CALIFORNIA STATE SCIENCE FAIR
2002 PROJECT SUMMARY

Name(s)
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Project Number
J1209

Project Title
Pi of Pieces: Unlimited

Abstract

Objectives/Goals
The objective is to derive recursive equations for Pi by estimating the area and circumference of a circle in terms of squares and triangles.

Methods/Materials
Method: If the radius of a circle = 1, the circumference = 2(PI), and the area = PI. Step #1: Inscribe a square in a circle of radius 1. Step #2: The sum of the four sides divided by two is the first estimation of Pi from circumference. The area of the square is the first estimation of Pi from area. Step #3: Draw a radius, bisecting a side of the square. Connect both ends of the side to the point where the radius intersects the circle. This forms an isosceles triangle with the side of the square as its base. Repeat this on the other 3 sides. Step #4: Determine the isosceles sides using the Pythagorean theorem. The sum of eight isosceles sides divided by two is the next estimation of Pi from circumference. Summing the area of the four triangles to the former area gives us the next estimation of Pi from area. Step #5: Using each isosceles side as the next base, repeat this procedure to get a new set of smaller triangles. Determine the next estimations for Pi. Step #6: Step 5 can be repeated endlessly: there will always be space above the sides of the last set of triangles. This produces equations of infinite terms with patterns for Pi. Step #7: From the patterns expressions for Pi can be written. This method can be applied with an inscribed triangle to obtain two more equations. Materials: Computer, printer, calculator, paper, pencil, eraser, ruler, compass, and protractor.

Results
The four equations I have derived are:
1. K(0) = -2; K(n) = sqrt(2 + K(n-1)), n >= 1; CPi(n) = 2^n * sqrt(2 - K(n)), n >= 1
2. APi(1) = 2; APi(2) = 2 * (sqrt(2) # 1); K(0) = -2; K(n) = sqrt(2 + K(n-1)), n >= 1; APi(n) = Pi(n-1) + 2^(n-1) * sqrt(2 - K(n-1)) # 2^(n-2) * K(n-1), n >= 3
3. K(0) = -1; K(n) = sqrt(2 + K(n-1)), n >= 1; CPi(n) = 2^(n-2) * 3 * sqrt(2 - K(n-1)), n >= 1
4. APi(1) = (3 * sqrt(3)) / 4; K(0) = -1; K(n) = sqrt(2 + K(n-1)), n >= 1; APi(n) = 2^(n-3) * 3 * sqrt(2 - K(n-2)), n >= 2

Conclusions/Discussion
I was able to find patterns and derive four different recursive expressions for Pi. I have written a computer program(C++) implementing the equations. I am planning to derive equations for other regular polygons, and format a unified equation.

Summary Statement
Using the approximations of the area and circumference of a circle, in terms of squares and triangles, four recursive equations for Pi have been derived.

Help Received
My father taught me the use of the Pythagorean theorem, and that when the radius bisects a chord in the circle they are perpendicular. He helped cut and paste things for the board, and stayed with me late nights encouraging me. My science teacher, Mrs. Terra, allowed the entire class to work on their projects in class