CALIFORNIA STATE SCIENCE FAIR
2003 PROJECT SUMMARY

Name(s)
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Project Number
J1203

Project Title
Simulated Billiard Ball Paths

Abstract

Objectives/Goals
My objective was to determine how the initial angle of an idealized billiard ball path, starting from a corner of a rectangular table with integral dimensions, affects whether the path will eventually end in a corner. Based on some mathematical background research, I hypothesized that the path will terminate in a corner if and only if the tangent of its initial angle is rational.

Methods/Materials
Using the Logo programming language and its turtle graphics facilities, I wrote a computer program to simulate rectangular tables and billiard ball paths launched from a corner. A test harness executed the basic simulation program for many different rational tangent angles and integral rectangle dimensions, recording the results in a file. This automated technique was made possible by the fact that all such paths did, in fact, terminate. Irrational tangent angle paths, on the other hand, required manual execution because such paths appeared never to terminate, making them unsuitable for automatic scheduling.

Results
I tested 63 different angles with rational tangents (systematically generated and with elimination of duplicates), and for each angle 100 rectangles of different integral dimensions, for a total of 6300 tests. In all of these, the paths terminated in a corner. I also tested angles with irrational tangents, such as 60°, 30°, and 50°, and found that such paths did not terminate in a corner. Because the tests in this second group never terminated, I could not run as great an abundance of them as in the rational tangent case.

Conclusions/Discussion
My hypothesis was correct. The paths with rational tangent angles terminated in a corner, whereas those with irrational tangent angles did not. While the simulation could only test a finite number of cases and was subject to the usual issues of numerical precision, my experiment sets the stage for confidently attempting to prove the mathematical statements of these outcomes.

Summary Statement
My project investigated, via computer simulation, the relationship between the initial angle of an idealized billiard ball path and the potential termination of the path in a corner.

Help Received
My father provided mathematical background that helped me to formulate my hypotheses. He also commented on my program and report. My mother helped with the layout of the display.