Name(s)

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Project Number

J1210

Project Title

The Secret of Nim: Mapping Finite Groups under Nim Addition to N-Dimensional Simplexes

Abstract

Objectives/Goals

The purpose of this project is to determine if there is a mapping between Nimbers (non-negative integers under Nim addition) and multidimensional objects called Simplexes, thus demonstrating the power of finite groups to relate very different mathematical objects to each other.

Methods/Materials

I researched the impartial mathematical game of Nim including Nimbers, the basics of Nim addition (binary addition without carrying), group properties, and the winning strategy for Nim as proven by Charles L. Bouton. I also researched Simplexes and their properties. I proved that Nim addition has the properties of an Abelian group and showed how Nimbers can be counted with their base 2 representations. I also determined a way to find the number of vertices, edges, and face-ns in a Simplex and proved that there is a one-to-one mapping between Nimbers and Simplexes. I then explored properties of this mapping to show how a Simplex could be used for Nim addition and how Nimbers determine their own unique Abelian groups and are locations in multidimensional space. I constructed a Simplex-3 using Zometool to illustrate this mapping. Materials used in this project are Zometool, a Dell PC running Microsoft Windows 98 and Word 97 and an HP printer.

Results

This project describes Nim addition as binary addition without carrying and shows how to use Nim addition to win a game of Nim. It demonstrates that Nim addition has the 5 Abelian group properties. Graphing Nimbers onto Simplexes is shown for lines, triangles, and tetrahedrons. The mapping of Nimbers to n-dimensional Simplexes is proved and illustrated using Pascal's triangle.

Conclusions/Discussion

The hypothesis that there is a mapping between Nimbers and Simplexes was proven. This required an analysis of Nim addition with proofs of its Abelian group properties. Combinatorics, one-to-one mappings, Pascal's triangle, and the binomial theorem were all utilized for the proof. Other observations include how to use the graph for Nim addition, and the fact that Nimbers are locations in multidimensional space and determine their own unique Abelian group.

Summary Statement

This project maps finite Abelian groups under Nim addition (binary addition without carrying) to multidimensional objects called Simplexes, demonstrating their use to win the game of Nim.

Help Received

My dad helped me research difficult topics and taught me how to set up a backboard. My mom made sure that the project could be understood and helped with the backboard layout. Mr. Sewell, my math teacher, suggested various corrections to my report.