CALIFORNIA STATE SCIENCE FAIR
2003 PROJECT SUMMARY

Name(s)
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Project Number
S1222

Project Title
Geometrical Representations of the 24th Order Permutation Group

Abstract

Objectives/Goals
The objective of this study was to determine if the 24th order permutation group can be represented by geometrical symmetry operations of rotation and reflection. A group is a set of elements and operators that have the following properties: (1) closure, (2), associativity, (3) identity, (4) (inverse). A permutation group is a group that has n factorial elements, where n is an integer representing the number of objects being permuted.

Methods/Materials
Four points were found in three-dimensional space with the property that each is equidistant from the other three. These four points can be made to coincide with four of the eight vertices of a cube or with the four vertices of a regular tetrahedron. Matrices were constructed to represent the twelve rotation operators and twelve reflection operators of the permutation group. These matrices have the property that when applied to the vertices of a regular tetrahedron, all 24 possible permutations of these four vertices are obtained.

Results
The researcher found that the 24th order permutation group can be geometrically represented by the four vertices of a three-dimensional regular tetrahedron. The symmetry elements are: one identity element, three two-fold rotations, eight three-fold rotations, and twelve reflections. Matrices were found to represent each of the 24 symmetry operations.

Conclusions/Discussion
There does indeed exist a three-dimensional geometrical representation for the 24th order permutation group. It is intriguing to contemplate the next logical step: a four-dimensional hyper-solid having five vertices, all of which are equidistant from the other four. If such an object exists it will have 120 symmetry operators associated with it. Sixty of these will be rotations in four-dimensional space and sixty will be reflections. This possibility will be studied in the project I will do next year.

Summary Statement
This project finds geometrical representations of the 24th order permutation group.

Help Received