Name(s)	Project Number
Rocio Mercado	**S1216**

Project Title

The Pappus Chain Theorem: A Proof

Abstract

Objectives/Goals
The Pappus Chain Theorem, a type of Euclidean geometry, was discovered and proven by Pappus of Alexandria in the third century. In the 1800's, Jacob Steiner, a Swiss mathematician, found a simpler proof for the theorem which used the method of circle inversion. The objective of this project is to research the Pappus Chain Theorem and circle inversion to use the new knowledge and proof the theorem myself.

Methods/Materials
1 drawing compass; 1 12" long straightedge; a protractor; color pencils; a computer w/ internet access (for research); library access/encyclopedias (for research)

Results
After applying the concept of circle inversion to the Pappus Chain, it was plainly visible that hidden between all the lines were similar triangles. The "triangles" are similar because of certain angles I found: $x(n)$, $y(n)$, $z(n)$, $x(n)$, $Y(n)$, $Z(n)$. The proof is much simpler than it appears, and the two triangles, along with circle inversion, have a big part in proving that $hn=n*d(Cn)$.

Conclusions/Discussion
From carrying out this experiment/proof, I conclude that the line creating a right angle from line AC to the center of inscribed circle n is equal to the diameter of circle n multiplied by n, $hn=n*d(Cn)$. I know this is true because the philosophy of circle inversion doesn't change when using it to corroborate with the Pappus Chain Theorem. During experimentation, I learned that this theorem is even true for n=0 and n=(forever), since the chain is a sort of fractal. At n=(forever), it will never touch line AC, therefore it will never touch point A and every circle has a centerpoint. If I was to carry out this project again, I would improve it by constructing better visuals with less human error in the constructions so that it will be easier for people to see the process and all the little details. That is also why I would make it bigger, but big compasses are hard to find. I would also use a computer program to make the constructions because by hand there is too much human error and it is very time consuming.

Summary Statement

The Pappus Chain Theorem, $hn=n*d(Cn)$, is an infinite chain of tangent circles inscribed inside an arbelos and can be proven using Jacob Steiner's method, circle inversion.

Help Received

Mr. Delaby (mentor)