CALIFORNIA STATE SCIENCE FAIR
2006 PROJECT SUMMARY

Name(s)
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Project Number
S1222

Project Title
On the Expected Winding Number of a Random Walk on the Unit Lattice

Abstract

Objectives/Goals
Some recent studies have focused on the winding number of a random walk. Given a random walk s starting at (1,1) on the unit lattice, the winding number w of s is the number of signed complete rotations the walk has made about (1/2,1/2). Despite the known results on the continuous winding number, the discrete version appears to be unstudied. This project investigates the root mean square expectation of the winding number.

Methods/Materials
We rephrase the problem in terms of a diagonal lattice and determine the winding number as a function of two variables counting steps beginning and ending on the positive x-axis. We then condition on the values of these variables and examine the change in expectation created by each additional step in the walk to express the desired expectation as a summation of only two smaller expectations. A symmetry that yields a bijection between types of these random walks allows us to determine these unknowns and thus reach our final result.

Conclusions/Discussion
We have found an explicit expression for the RMS expected winding number after n steps of a random walk beginning at (1,0) on the unit lattice. This expression is in terms of a binomial sum; we first find the expectation recursively and then exploit a symmetry of random walks to solve the recursion. This result gives us a better understanding of the rotational properties of random walks and thus may be useful in further investigations into this field.

Summary Statement
My project determined the exact value of the expected value of the winding number, the number of rotations that a random walk, or a random path, on the unit lattice makes around a point.

Help Received
Was mentored by Mr. David Pritchard, a graduate student at MIT.