Name(s)

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S1223

Project Title

The Solution of the Dirichlet Problem with Rational Boundary Data

Abstract

Objectives/Goals

I study the Dirichlet problem for the Laplace operator on arbitrary simply connected bounded domains in the plane with rational holomorphic boundary data. My first goal is to answer the following question: When are all solutions rational? My second goal is to find an explicit formula for the solution.

Methods/Materials

I form and prove several theorems related to my question, and combine them into a single theorem, which among other things, completely answers my question. By extending the proof of my theorem, I obtain an explicit formula for the solution on any simply connected bounded domain in the plane.

Results

My main results include the following:

1. The solution to every Dirichlet problem with rational holomorphic data is rational if and only if a Riemann map is rational.
2. The solution to every Dirichlet problem with rational holomorphic data is rational if and only if the Bergman kernel of the domain is rational.
3. The solutions to the above Dirichlet problems are rational if and only if the solution is rational for a single, relatively 'simple' data function, namely, the function 1/(z-a), where a is a point in the domain.
4. The Bergman kernel K(z,w) is rational if and only if K(z,a_1) and K(z,a_2) are rational, where a_1 and a_2 are any two points in the domain.

Furthermore, I state and prove an explicit formula for the solution to any such Dirichlet problem in terms of only two functions: a Riemann map of the domain and the boundary data function.

Conclusions/Discussion

My theorem completely answers my question both geometrically and algebraically; namely, in terms of a Riemann map and in terms of the Bergman kernel of the domain. My theorem also connects the concepts of the Dirichlet problem, the Riemann map, the Bergman kernel, and rational functions in a simple way. Furthermore, my formula is the first explicit solution to the Dirichlet problem on arbitrary simply connected bounded domains in the plane.

Summary Statement

I completely characterize, both geometrically and algebraically, all domains in the plane in which the solution of every Dirichlet problem with rational holomorphic data is rational, and obtain the first explicit formula for the solution.

Help Received

My mentor, Prof. Peter Ebenfelt of UCSD, was available to answer my questions as I worked through proofs. I completely wrote the paper on my own, and Prof. Ebenfelt looked over it and made suggestions for improvement.