CALIFORNIA STATE SCIENCE FAIR
2007 PROJECT SUMMARY

Name(s)
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Project Number
S1309

Project Title
A Comparison of Numerical Integration Approximations: Midpoint, Trapezoidal, and Simpson's

Abstract

Objectives/Goals
The objective of this project is to determine if the Midpoint Approximation, the Trapezoidal Approximation, or the Simpsons Approximation would give the most accurate result for the area under the curve.

Methods/Materials
1.Take the following functions: y=2x-1, y=-3x+2, y=1/2(x-2)^2+1, y=-(x+1)^2+4, y=cos(x), y=sin(x), y=2e^(0.5x), y=0.5e^(-0.5x), y=1+log(x), and y=ln(2-x).
2.Calculate the actual area under the curve for each function using integration.
3.Divide each curve into 2, 4, 8, 16, and 32 equal subintervals.
4.Using the Midpoint, find the approximate area under the curve for each subinterval.
5.Using the Trapezoidal, find the approximate area under the curve for each subinterval.
6.Using the Simpsons, find the approximate area under the curve for each subinterval.
7.Record the results and compare them to the actual area under each curve.
Note: Programs on TI-89 calculator and Java programs are used to compute the area.

Results
For the linear functions, the areas were always exact regardless of the methods used. In the case of quadratic functions, the Simpsons method gave the best approximation and the Trapezoidal provided the worst. Next, for the trigonometric functions, the Simpsons gave the most accurate approximation while the Trapezoidal gave the least accurate approximation. Then in the case of exponential functions, again the Simpsons gave the best approximation and the Trapezoidal yielded the worst approximation. Finally, for the logarithmic functions, the result was the same; the Simpsons offered the closest value to the actual area and the Trapezoidal gave the least accurate result.

Conclusions/Discussion
In most cases I was correct that the Simpsons gave the most accurate approximation. However, I incorrectly predicted that the Trapezoidal would do better than the Midpoint. The Midpoint gave more accuracy than the Trapezoidal. Next, I verified my intuition that as the number of subintervals increases, the approximations get better. In fact, the Midpoint can achieve the accuracy of the Simpsons at very large n. Also, I found that error in the Trapezoidal is almost twice the error in the Midpoint, bur in opposite direction. Another interesting thing with the Simpsons is that its accuracy improves dramatically over n. Finally, I conclude that, regardless of the function, the approximation techniques work better if the curvature of a function is not too high.

Summary Statement
My project explores to find if the Midpoint, Trapezoidal, or the Simpsons would give the most accurate result and analyze the data for comparison between the three methods.

Help Received
My father introduced me to the analytic method of evaluating definite integrals in calculus and showed me how to calculate the area under a curve using integration. He also helped me with the programming on the TI-89 calculator.