CALIFORNIA STATE SCIENCE FAIR
2008 PROJECT SUMMARY

Name(s)

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Project Number

S1611

Project Title

Investigation of Mathematical Models of Harmonic Spring Motion

Abstract

Objectives/Goals

When a mass suspended vertically by a spring is pulled, it moves up and down in a simple way. This periodic motion can be described mathematically in terms of a simple sinusoidal equation. In the real world, however, resistive forces are always present and cause the mass to slow down over the time period. This effect is called damping. The objective of this project is to investigate the mathematical models of harmonic spring motion with and without damping and to study the effect of increased damping.

Methods/Materials

Find a general solution to the mathematical model for undamped harmonic spring motion. Set up spring-mass system and measure spring constant. Pull the mass down and release so that oscillation is up and down. Using CBR 2 and TI-89 with Ranger program, collect data for several cycles. Find a general solution to the mathematical model for damped harmonic spring motion. Add a damping effect to the motion and collect data in a similar manner. Derive the exact solutions using the experimental data. Plot both the experimental and predicted (mathematical model) and compare. Increase damping constant and observe the effect to the solution.

Results

The average spring constant is measured at 23.1 N/m. The general solution of the undamped spring motion model is $y(t) = A\cos(wt + b)$ and its mathematical model is $y(t) = 0.138\cos(6.481t - 3)$. For the damped spring motion model, the general solution is $y(t) = A\mathrm{Exp}[-ct/(2m)]\cos(wt + b)$ and its mathematical model is $y(t) = 0.21\mathrm{Exp}(-0.2t)\cos(5.7t - 9.5)$. Both experimental data and model functions are plotted for analysis. The critical damping constant is 7129.

Conclusions/Discussion

My hypothesis that the mathematical model for undamped spring motion would better approximate the experimental data when compared to the mathematical model for damped spring motion was proven incorrect. My results show that both models appear to approximate the experimental data during the first several cycles, but lose their accuracies over time with little error differences between the two models. Assumption of no damping accounts for the errors in the undamped spring model, and for the damped model, the damping coefficient is the deciding factor. The rate of exponential decay of the measured data is faster than the model. Finally, the sinusoidal oscillatory behavior fades out fast as damping constant is increased and eventually is replaced with steep exponential decay to zero.

Summary Statement

This project is to derive the mathematical models of harmonic spring motion using the experimental data and compare to determine how closely the model functions approximate the actual data.

Help Received

Wolfram Research, Inc. generously offered a trial version of their software Mathematica for this project.