

CALIFORNIA STATE SCIENCE FAIR 2009 PROJECT SUMMARY

Name(s)

Dean Hathout

Project Number

J1604

Project Title

Tricky Triangles: A Probability Problem with Theoretical Solution and Monte Carlo Simulations

Abstract

Objectives/Goals The objective of this project is to study a challenging probability problem: if a stick is cut in two places at random to form three pieces, what is the probability that a triangle can be constructed from those pieces? This problem is solved theoretically using probability theory, and the solution is compared to empirical experiments and Monte Carlo simulations. The usefulness of Monte Carlo simulations is further explored using variants of the problem that do not have an easy theoretical solution.

Methods/Materials

Geometric probability theory is used to solve the problem. Sixteen 10 inch dowels and a paper cutter were used to conduct direct experimentation. A Dell PC with Matlab Software was used to program a Monte Carlo simulation of the problem (N=100,000) and was run four times. Simulations for problem variants were also programmed.

Results

The theoretical solution gives a probability of 25%. This showed an excellent match with both the experimental data and Monte Carlo simulations, which gave the same results. Monte Carlo simulation results of the problem variants are also obtained.

Conclusions/Discussion

This experiment explores the three ways to study a probability problem: theoretically using probability theory, empirically using repeated direct experiments, and using Monte Carlo simulations. For the problem posed above, there was excellent agreement between the theoretical, empirical, and simulation results. The project also shows the usefulness of Monte Carlo simulations for variants of the problem that do not have easy theoretical solutions.

Summary Statement

This project explores a challenging probability problem using theoretical, empirical, and Monte Carlo simulation approaches; furthermore, it demonstrates the usefulness of Monte Carlo simulations in more complex problem variants.

Help Received

Mom helped with use of paper cutter. Dad helped load Matlab software on laptop for presentation.