CALIFORNIA STATE SCIENCE FAIR
2009 PROJECT SUMMARY

Name(s)

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Project Number

S1607

Project Title

Number Theory Meets Algebra: Walking the Path of Fermat, Euler, and Gauss

Abstract

Objectives/Goals

A (binary) quadratic form is a polynomial f(x,y) = ax^2 + bxy + cy^2, where a, b, and c are integers. A number n is represented by a quadratic form f(x,y) if n = f(x,y) for some integers x, y. The investigations of Fermat, Euler, Gauss, and many other great mathematicians led them to discover the patterns governing the multiplication of two numbers represented by the same quadratic form.

My objective is to discover a method to multiply two numbers represented by the same quadratic form f.

Methods/Materials

To demonstrate the process, I will play a card game called the Quad game which contains some big squares, small squares and rectangles. The game is a pictorial representation of a quadratic form in which the big square is x^2, the small square is y^2, and the rectangle is xy. The object of the game, given a particular hand, is to create one square of some large size plus optionally several squares of a single smaller size. The game is subject to certain rules which I will also explain. The winning strategies for this game provide the necessary clues to derive the multiplication formula for quadratic forms.

Results

Using the Quad Game, I showed how to multiply two numbers represented by the same quadratic form f. My main result is that their product is also represented by some quadratic form g. In some cases g is different than f, and in some cases g is the same as f.

Summary Statement

A method to multiply numbers represented by quadratic form(s) and derive a general formula for the same.

Help Received

I discussed my methods with Dr. Laurens Gunnarsen