CALIFORNIA STATE SCIENCE FAIR
2009 PROJECT SUMMARY

Name(s)

Leo Zhou

Project Number

S1625

Project Title

The Effects of Inverse Fourier Transform on Gaussian Random Fields

Abstract

Objectives/Goals

This project seeks a correlation between deterministic functions of standard deviation used to generate Gaussian random fields and the functions or surfaces generated by inverse Fourier transforming the fields.

Methods/Materials

I employed a personal laptop with Matlab for computation and Microsoft Excel for data processing. I experimented with various functions of standard deviation and graphed them, their inverse Fourier transforms, the Gaussian random fields generated, and the functions or surfaces generated. The number of crests is determined by scanning every point on the function or surface and check if adjacent points have lower values. Linear regression analysis at the crests of two functions or surfaces is used for quantitative analysis of the correlation.

Results

It is observed that the generated functions or surfaces exhibit predictable patterns given a function of standard deviation, and these functions or surfaces resemble the inverse Fourier transform of the standard deviation function considerably at their extrema. Further testing and analysis demonstrate a linear correlation between the two near their extrema with high statistical significance. They correlate very significantly ($|r|>=0.9$) in high frequencies, typically 90%, in cases of smooth or oscillating waves. However, the frequencies of highly significant correlations can be less than 10% for cases of chaotic waves, but fairly significant correlations ($|r|>=0.7$) are still common. Besides, the frequency of significant correlations depends on the predictive range; and for individual extremum, sometimes $|r|>=0.99$ despite low frequencies.

Conclusions/Discussion

As supported by my results, there indeed exist a strong correlation between deterministic functions of standard deviation and the generated functions and surfaces via the inverse Fourier transform of the standard deviation function. This correlation is very significant in applicable cases and can be used to effectively classify and to predict the behaviors of arbitrary functions and surfaces, especially those in error control, signal analysis, geophysics and ocean science.

Summary Statement

My project analyzes behaviors of waves and surfaces via examining the consequences of inverse Fourier transforming Gaussian random fields, and concludes with a scheme to classify and predict arbitrary waves and surfaces.

Help Received

My 2009 Canada/USA Mathcamp advisor answered some of my questions.