Name(s)
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Project Number
S1415

## Project Title

## Are All Irrational Numbers Equally Irrational?

## Objectives/Goals

Abstract
The purpose of my project is to investigate whether or not all irrational numbers contain patterns within their continued fraction form. Additionally, if they do have patterns, are there patterns amongst the patterns themselves?

## Methods/Materials

1. Choose irrational numbers to test. I chose the irrational numbers pi, e, the square roots of 2 through 100 (excluding the perfect squares in between) and the cube roots of 2,3 , and 4 .
2. Next, express these irrational numbers in continued fraction form.

To do this, start by entering the irrational numbers in a calculator. The calculator will give a trailing decimal approximation.
Subtract the integer portion from the number, and record that integer as the first data value. For example, in (approximately 3.14159\#), subtract 3 from the calculator\#s approximation to get . 14159...
Next, perform 1 divided by the result. Continuing with our example, we do 1 divided by $.14159 \#$ to get 7.06251\#
3. Repeat step 2 at least 9 more times and record the results. In our example, the next integer portion to subtract and record is 7 .
Record all of the results calculated in our list. In our pi example, the list would look like:
3,7,15...
4. Look for patterns in and amongst the lists formed.

## Results

Expressing irrational numbers in continued fraction format does indeed reveal startling patterns. For example, the square root of 17 , which seems to be a nondescript, endless number (4.123105626\#) in decimal format reveals itself to be quite amazing in continued fraction format with its simple pattern of all 8s. Even more, it appears that every square root has a pattern within it when expressed in continued fraction format. Nevertheless, it is clear there are some other irrational numbers that still don\#t have patterns to them- such as or the cube roots of 2-4.

## Conclusions/Discussion

My hypothesis was partially correct. In my hypothesis, I thought that all irrational numbers still wouldn\#t exhibit any patterns in continued fraction form. This was true for some irrational numbers, such as and the cube roots of 2-4, but was untrue for all of the square roots that I tested. Even more, I discovered some startling patterns inherent amongst the patterns of the square roots themselves.
Summary Statement
Expressing irrational numbers in continued fraction format can reveal simple patterns.

## Help Received

Father helped show how to write numbers in continued fraction format

