

### CALIFORNIA STATE SCIENCE FAIR 2015 PROJECT SUMMARY

Name(s)

Jared A. Tramontano

Project Number

# S1428

#### **Project Title**

## **Fuzzy Structures with Application to Differential Topology, Manifold Learning, and Specialized Concepts**

#### Abstract

**Objectives/Goals** The main purpose of the project is to introduce a notion of Differential Topology in the setting of Fuzzy Set Theory, introduced by Zadeh in the 1960's. As Fuzzy Set Theory is just a generalization of set theory, Fuzzy Differential Topology is just a generalization in the normal Euclidean setting. All concepts were developed with application to Manifold Learning in mind. Namely, one may consider fuzzy data manifolds in computer science.

#### Methods/Materials

The only materials needed for this project were a blackboard and chalk, various textbooks (notably Differential Topology by Hirsch), and academic papers.

#### Results

The standard setting of Differential Topology was extended to the notion of Fuzzy Topological Vector Spaces. Although slight attempts had been made previously, such attempts lacked rigor and depth. The new notions developed include higher dimensional fuzzy sets, Fuzzy Topological Separation Axioms, Fuzzy Differentiation, Fuzzy Atlases, Fuzzy Tangent Bundles, Fuzzy Cotangent Bundles, and Fuzzy Lebesgue Integration.

These notions give rise to a proper foundation for Fuzzy Differential Topology, where most concepts, such as homology, should have suitable analogies in these spaces.

#### **Conclusions/Discussion**

In terms of computer science, these notions have been implemented in algorithms in manifold learning that seem to better handle real-world data sets, using the notion of fuzzy set theory. Two students at MIT have handled the writing of said algorithm, and the three person paper detailing my notions of Fuzzy Differential Topology and their algorithms, will be submitted for publication this Fall.

In summary, proper generalizations of Differential Topology have been established, thus setting the foundations for future work in the area.

#### **Summary Statement**

Combining concepts from Differential Topology and Fuzzy Set Theory.

#### **Help Received**

Discussions with various professors at UCR and UCI, as well as students of MIT and Harvey Mudd.