CALIFORNIA STATE SCIENCE FAIR
2017 PROJECT SUMMARY

Name(s)
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Project Number
S1514

Project Title
Factorization of Recurrence Relations

Abstract

Objectives/Goals
The combinatorial solution to the recurrence relation a_{n} =a_{n-1} +a_{n-3} +a_{n-4} leads to the trend, a_{2n} =f_{n}^2 , where f_{n} is the n-th term in the Fibonacci sequence. This project explores the generalization of the solution to this recurrence relation to yield a new family of sequences where every k-th term is the k-th power of u_{n}: a_{kn} =u_{n}^k. u_{n} is the solution to Horadam#s second order recurrence relation of the kind u_{n}=p#u_{n-1} +q#u_{n-2}, where p, q are integers.

Methods/Materials
I used several techniques to investigate the factorization of these recurrence relations. First, I used diagrams to illustrate the factorization. I explored how to find the recurrence relation with even terms leading to square of generalized second-order recurrence relation through bijections and Binet-like formulas. I took the trend, described above, of squares of Fibonacci numbers for every even term and generalized it for cubes of any second-order sequence. Finally, I derived a generalized version of this sequence, with every k-th term yielding the k-th power of the generalized second-order sequence. Techniques that I used drew from number theory. I used Hadamard products, Cauchy#s residue theorem, diagrams, Binet#s formula, partial fractions, and work by Hoggart and Legendre. An understanding of recurrence relation and generating functions was paramount, as well.

Results
The solution to the recurrence relation was found to be a_{2n} =f_{n}^2 and a_{2n+1} =f_{n}f_{n+1}. The bijection for a_{2n} was denoted by the number of ways we can tile two rectangles of length 1xn with 1x1 square and 1x2 rectangle. This bijection was generalized to k rectangles for a_{kn} and a solution was found for its generating function through bijection as well as Binet-like formula.

Conclusions/Discussion
These results represent the product of a year of investigation, however, additional work is being done to explore related problems in this field, such as examining similar families for Catalan and Motzkin numbers.

Summary Statement
This project derives with the recurrence relation, generating function and bijection for a new family of sequences, where the k-th term a_{kn} =u_{n}^k, where u_{n} is the n-th term of Horadam#s generalized second order sequence.

Help Received
All work on this project was done by me at my home. This project was derived from a problem provided by Dr. Simon Rubinstein-Salzedo and periodically offered input when requested.