

CALIFORNIA SCIENCE & ENGINEERING FAIR 2018 PROJECT SUMMARY

Name(s)

Rachana Madhukara

Project Number

S1510

Project Title Asymptotics of Character Sums

Abstract

In this project, we aim to prove certain properties about a particular function $c(n) = b_nr(n)$. This is where b_n is a Boolean function with b_n being 1 if $n = x^2 + y^2$ for some integers x and y or 0 otherwise and $r_chi(n)$ is the sum of all of the Dirichlet characters of d, where d divides n. The function c(n) sums the all of the chi values of the divisors of a certain number n if and only if n can be expressed as the sum of two squares. Therefore, the question we ask is the following: What are the asymptotics of the character sums of the function c(n)?

Results

Objectives/Goals

In order to investigate this problem, we first represent the character sum of r(n) as an asymptotic and prove that the asymptotic is roughly L(1, chi) with a small error term. Additionally, we compute a representation for the character sum c(n) as an Euler product, and also find error bounds on the asymptotic for the character sum.

Conclusions/Discussion

We analyzed the asymptotic, or growth rate, of a very special function c(n) which describes a very particular group of primes. In specific, our growth rate describes the group of primes which are dependent on two character values. Additionally, we found some error bounds on how accurate our asymptotic is.

Summary Statement

In this project, an asymptotic for a function c(n) was found along with an error term using elementary number theory techniques.

Help Received

My mentor Dr. Simon Rubinstein-Salzedo greatly helped me with this project. His main role in the project was always leading me in the right direction. He did this by providing me with lots of relevant papers to read and giving me ample suggestions.