CALIFORNIA SCIENCE & ENGINEERING FAIR
2019 PROJECT SUMMARY

Name(s)

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Project Number

S1401

Project Title

A Novel Analytical Approach to Determining Parameter Derivatives of an Object under Motion Constraints

Abstract

Objectives

When the path of an object is described by the vector equation in the form of r(u,v,w...)=<f(u,v,w...),g(u,v,w...),h(u,v,w...)>, the expression ((d^n)r)/(d(p^n)), where p is a variable that represents one of the parameters u, v, w..., can be used to find the velocity and acceleration of the object with respect to the parameters at any position. However, when this parameterization is unknown, and the object moving along a certain path is subjected to various constraints, a different approach must be employed to find the velocity and acceleration of the object. The objective of this project is to find a generic formula for the acceleration with respect to the parameters of an object in such a situation, and contrast the efficacy of this analytical approach with two existing approaches, curvature and numerical approximation of differential equations.

Methods

The project involved first deriving the new mathematical approach to solve for acceleration, and extending the derivation to three dimensions and to a broader variety of curves and constraints. As such, simply pencils, paper, and a calculator were used. The analytical method was subsequently tested against existing approaches of numerical approximation of acceleration with the use of Euler's Method, and calculation of acceleration using curvature. Comparisons were made based on the metrics of computational efficiency, accuracy, and viability in real world applications. Tests were carried out by implementing these solutions in code using Java and the Apache Commons Math library.

Results

The results of the tests indicated that the new analytical approach fared considerably better than the other approaches on all criteria tested. Its computational efficiency was approximately 22.5% better than that of the curvature approach, and 97.0% better than that of the Euler's Method approach, and produced results of greater accuracy. The analytical approach's algorithmic complexity also proved to be lower than that of the curvature approach, and significantly less than that of the O(n^2) Euler's Method approach.

Conclusions

With faster runtimes and a 100% accuracy, this new approach could have wide applications in flight path planning, space path planning, and autonomous navigation, as it gives the ability to conduct real time calculations on how an object can traverse a certain path under various constraints. The acceleration metrics obtained with the analytical approach can ultimately be converted to engine power and orientation that these objects need to traverse the path. Further work planned includes developing a simulation using this equation for real time flight planning and potentially autonomous driving.

Summary Statement

Derived a new analytical approach to finding acceleration of an object subject to constraints on parameter derivatives and contrasted its viability with current existing methods.

Help Received

I worked on this project individually at home. I referenced a variety of sources for research material.