CALIFORNIA SCIENCE & ENGINEERING FAIR
2019 PROJECT SUMMARY

Name(s)

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Project Number

S1408

Project Title

Dynamics of the Tangent Map

Abstract

Objectives

Analyze the distribution of the iterations of the tangent map for most starting values, both measure-theoretically and topologically. Characterize sets of starting values whose trajectories avoid certain intervals.

Results

I proved that almost all points, in both measure-theoretic and topological senses, have dense orbits under iterations of tan(x). The next theorem proved that any pattern of behavior is possible and happens for exactly one point. Furthermore, the set of starting points whose trajectory avoids a given set of intervals is a Cantor set.

Conclusions

I developed a new method to analyze expanding maps with indifferent fixed points. The methods I used are simpler than what exists in the literature and could be used in a broader class of maps with indifferent fixed points. My project provides a relatively simple approach to a complicated problem studied extensively today. This approach can be used to study models with intermittent chaos, a topic of particular interest in physics, computer science, and certain areas of mathematics.

Summary Statement

I found a new method to study expanding maps with indifferent fixed points and applied it to prove four theorems regarding the density and patterns of the iterations of the tangent map.

Help Received

I discovered and proved all the theorems myself. After I completed my project, Prof. A. Gorodetski from UCI showed me prior research done on this topic and commented on my paper.