CALIFORNIA STATE SCIENCE FAIR
2003 PROJECT SUMMARY

Name(s)
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Project Number
J1207

Project Title
Endless Snowflake: Constructing Shapes with Infinite Perimeters and Finite Areas

Abstract

Objectives/Goals
To determine whether it is possible to construct shapes with infinite perimeters but finite areas.

Methods/Materials
Using the Geometer's Sketchpad program and a Koch curve, I began with an equilateral triangle and grew a snowflake through various generations which, if taken to infinity, would have an infinite perimeter but a finite area. Using power series, I calculated what that area would be. I then decided to develop a new fractal curve based on a square shape rather than the traditional triangular Koch curve.

Results
I calculated that this fractal pattern for the triangle would produce a figure of infinite perimeter, but whose area is only 1.6 times the area of the original triangle. Meanwhile, for the square, the area for an infinite perimeter shape would be 2.0 times the area of the original square.

Conclusions/Discussion
Ordinarily, when shapes are magnified, area grows faster than perimeter. However, using the idea of convergent series, it is possible to add ever-smaller increments of area such that while the perimeter grows to infinity, the sum of the areas remains finite. It may be possible to generalize this approach to three dimensions, producing a shape of infinite surface area and finite volume.

Summary Statement
My project uses the ideas of fractals and power series to construct a Koch snowflake, and to explore new families of curves with infinite perimeters and finite areas.

Help Received