CALIFORNIA STATE SCIENCE FAIR
2003 PROJECT SUMMARY

Name(s)

Daniel A. Whisler

Project Number

S1228

Project Title

One:Four: Generalizing an Area Ratio for Related Quadrilaterals

Abstract

Objectives/Goals

Nearly 2,000 years ago, Hero related a formula for the area of any triangle. Brahmagupta's generalization included the areas of cyclic quadrilaterals. Bretschneider's theorem further encompassed any quadrilateral. In the spirit of these mathematicians, an area ratio for two isosceles trapezoids--drawn such that the smaller polygon is formed using two adjacent vertices and two diagonal midpoints of the larger trapezoid--will be generalized for any quadrilateral.

Methods/Materials

Proving the one to four area ratio for any trapezoid was accomplished through geometric relationships. Using trigonometry, the proof was extended into a formula for any convex quadrilateral where area is 1/2 times the diagonal p times the diagonal q times the sine of any intersecting diagonal angle ß, A=1/2pqsinß.

Results

Since the smaller quadrilateral has diagonal lengths exactly one half the diagonal lengths of the larger quadrilateral, multiply both lengths together and its area is always one fourth of the total area. Should the diagonals not intersect, however, (possible in highly complex concave polygons) then the area formula is undefined.

Conclusions/Discussion

To address the concave polygons, this research report generalized the area formula even further: for any infinitely concave polygon, there exists an infinitely larger convex polygon whereby both areas are related through the equation A=1/4^n(1/2pqsinß), where n is the number of successive quadrilaterals constructed through the midpoints of diagonals p and q intersecting at an angle ß. Should n=1 for related quadrilaterals, then a one to four area ratio always exists.

Summary Statement

Generalizing the area for any four-sided concave or convex polygon in relation to another polygon.

Help Received

Mr. J. Briggs provided the computer software program and helped edit the research report.