CALIFORNIA STATE SCIENCE FAIR
2004 PROJECT SUMMARY

Name(s)

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Project Number

J1210

Project Title

Tiling with Shapes and Tessellations in Nature

Abstract

Objectives/Goals

The overall objective of this project was to explore which shapes can (or cannot) tile a rectangular grid or infinite plane (in one or multiple ways) and understand why.

Methods/Materials

We discovered many different types of amazing shapes that have been used in tiling in man-made objects such as puzzles, art, and architecture, and tessellations in nature such as crystals and honeycombs. We decided to experiment with the following shapes: Pentominoes, Heptiamonds, Wheelbarrow, Kite #n# Dart Pair, and Regular Convex Polygons (including Triangles, Squares, and Hexagons).

We built pentominoes from legos, heptiamonds from pattern blocks; wheelbarrow and kite #n# dart from tagboard, and different types of (triangular, square, and hexagonal) honeycombs with manipulatives. We experimented tiling appropriate rectangular grids and planes using these shapes.

We used a symmetry principal to reduce the number of tiling problems for pentominoes.
We counted the number of sides used in building honeycombs and noticed patterns for which we derived formulas for the amount of wax used.

Results

We found that we can tile some rectangular grids (8x8 with a square removed anywhere, 3x20, 4x15, 5x12, and 6x10) with twelve pentominoes. Additionally, we discovered that all but one of the 24 heptiamonds could tile a plane individually. We invented interesting, non-trivial ways of tiling a plane using both wheelbarrow and kite #n#dart pairs. Finally, we used manipulatives, algebra, and geometry to prove that hexagons are the most efficient shape (consume the least wax to create the same amount of area) from all regular convex shapes to build a honeycomb.

Conclusions/Discussion

We discovered that although many different types of convex and non-convex shapes (e.g. pentominoes, wheelbarrow) and several interesting combinations of them (e.g. kite n#dart) can be used to tile a plane in interesting ways, there are many simple shapes that cannot be used to tile the plane (e.g. a V-shaped heptiamond or a pentagon). Although we proved that hexagons use the least amount of wax in comparison to squares or triangles, we did not prove that they are better than irregular or multiple shapes to hold honey (we discovered that the general proof was given only five years ago in 1999!).

Summary Statement

In this project we explored which shapes can or cannot tile an infinite plane or rectangular grid, explained why, and applied our findings to nature and society.

Help Received

Parents - Provided overall guidance, helped develop project, and gave overall tips on display; Math Teacher - Helped on background research; Science Teacher - Gave project and display tips; Neighbor - Gave project and display tips; Science Fair Coordinator - Gave display tips