CALIFORNIA STATE SCIENCE FAIR
2004 PROJECT SUMMARY

Name(s)

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Project Number

J1212

Project Title

The Sequel of Nim: Symmetries and Transformations of n-Cubes and the Nimber-Simplex Graph

Abstract

Objectives/Goals

This project is a continuation of last year's project which mapped finite groups under Nim addition to n-dimensional Simplexes, creating the Nimber-Simplex graph. The goals of this year's project are first, to apply finite group theory to regular convex polytopes, second, to determine the symmetry group of an n-cube using the Nimber-Simplex graph, and third, to illustrate a reversible transformation between the Nimber-Simplex graph and an n-cube.

Methods/Materials

In this project, two isomorphisms are defined: one between the Nimsum group 2^n-1 and the Cartesian product C(2)^n, the other between the symmetry group of a Simplex-(n-1) and the permutation group S(n). The symmetry group of an n-dimensional hypercube is determined by mapping the group C(2)^n to the vertices of the n-cube as well as to diagonal matrices representing reflection operations. Permutations of coordinate axes, P(n), are shown to be isomorphic to S(n). The group of symmetries of the n-cube, G(n), is then a semidirect product of the normal subgroup N(n), representing the reflection symmetries, and the subgroup P(n). That is, G(n)=N(n)xP(n). A reversible transformation between the Nimber-Simplex graph in (n-1) dimensions and an n-dimensional hypercube is demonstrated. Materials used in this project include Zometool, an hp Deskjet printer, and a Dell PC running Microsoft Windows 98 and Word 97.

Results

The original ideas developed in this project include the definition of the Nimber-Simplex graph, the isomorphism between the Nimsum group 2^n-1 and C(2)^n, and mapping C(2)^n to vertices of an n-cube as well as the reflection matrices of the n-cube. These two results were used to prove that the Nimber-Simplex graph in (n-1) dimensions determines the symmetry group of an n-cube and that the Nimber-Simplex graph unfolds into an n-cube. Finally, it was discovered that this year's project connects all regular convex polytopes in n>4 dimensions!

Conclusions/Discussion

The first hypothesis that the symmetry group of an n-cube is a semidirect product of the symmetry group of a Simplex-(n-1) and the Nimsum group 2^n-1 was proven. The second hypothesis that there is a reversible transformation between the Nimber-Simplex graph in (n-1) dimensions and an n-cube was also proven. Since n-cubes and n-dimensional cross-polytopes are dual, the Nimber-Simplex graph relates all regular convex polytopes in n>4 dimensions.

Summary Statement

This project determines the symmetry group of an n-cube using the Nimber-Simplex graph, demonstrates a reversible transformation between the Nimber-Simplex Graph and an n-cube, and relates all regular convex polytopes in >4 dimensions.

Help Received

My father helped teach me group theory and guided my research of symmetries and regular polytopes. My parents assisted with backboard construction and reviewed the report for readability and technical accuracy. My math teacher acted as an advisor.