



CALIFORNIA STATE SCIENCE FAIR 2005 PROJECT SUMMARY

Name(s) Rebecca E. Jacobs	Project Number J1209
Project Title Simplex to Complex: From the Nimber-Simplex Graph to Codes, Lattices, and Groups	
<p style="text-align: center;">Abstract</p> <p>Objectives/Goals This project is about the relationship between the Nimber-Simplex graph, error-correcting codes, lattices, and finite simple groups. My previous projects defined the Nimber-Simplex graph (NSG) as a map between finite groups under Nim addition and n-dimensional simplexes, then linked the NSG in n-1 dimensions to n-cubes. The goals of this year's project are to explore coding theory and sphere packing with the specific hypotheses that the NSG can be used to construct binary error-correcting codes, lattices, and finite simple groups.</p> <p>Methods/Materials In this project, a relationship between binary linear codes and the NSG is shown, specifically that the NSG retains its fundamental properties when used as the word space and codeword space of a binary linear code. The NSG is shown to be closely related to Hamming codes, and the Ham(3) and Golay G24 codes are constructed using the graph. The relationships between lattices and codes are discussed, and the D3 and E8 lattices are constructed using n-cubes, the $Z(n)$ lattice, and codes. The 24-dimensional Leech lattice is constructed from the G24 code. The NSG is defined as a Steiner system, and a particularly nice isomorphism between the automorphism group of the NSG and $GL(n, 2)$ is shown. Lastly, the structure of the automorphism group of the Leech lattice is described in order to show the relationship of the project to finite simple groups.</p> <p>Results This project represents a unique approach to coding theory and sphere packing. Original contributions include the construction of the Ham(k) codes from the NSG, the use of the NSG as the word and codeword space of a binary linear code, the NSG as a Steiner (2, 3, 2^{n-1})-system, and the isomorphism between the NSG's automorphism group and $GL(n, 2)$.</p> <p>Conclusions/Discussion The three hypotheses of this project were proven: the Ham(k) and G24 codes can be constructed using the Nimber-Simplex graph; the Leech lattice, as well as the E7, E8, and $Z(n)$ lattices, can be constructed using the graph; and certain simple groups associated with the Leech lattice's automorphism group can be constructed using the graph.</p>	
Summary Statement This project links the Nimber-Simplex graph to error-correcting coding theory, lattices, and finite simple groups.	
Help Received My father helped me learn sphere packing and coding theory. My parents assisted with backboard construction and reviewed the report for readability and technical accuracy. My math teacher acted as an advisor.	