CALIFORNIA STATE SCIENCE FAIR
2012 PROJECT SUMMARY

Name(s)
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Project Number
S1401

Project Title
On the Theory of Functions and Collatz-Like Conjectures

Abstract

Objectives/Goals
The modern field of mathematics is a widely varied and diverse subject, with subfields ranging from statistics to homological algebra. Due to this variety, different tools have been developed in each area, which means that discoveries in one area rarely give insights into other areas. This project#s original goal was to find a structure which could be used to algebraically study a seventy-five-year-old conjecture known as the Collatz Conjecture, but soon expanded to the goal of uniting all of mathematics, which would provide a a powerful new tool for making progress on any mathematical problem.

Methods/Materials
This project focuses on the new idea of a functionally-closed set, which consists of a semigroup of functions (a set of functions which is closed under composition) that map a given set to itself, along with the set itself. The inclusion of the set itself leads to novel notions of when two functionally-closed sets are #homomorphic#, or have similar structures. The resulting structure, to the best of the author#s knowledge, has not been studied previously.

Results
After fleshing out the basic theory of these structures, a way was found to map almost any mathematical structure onto certain functionally-closed sets, and vice versa. This provides a framework by which one can transform problems in one area of mathematics, such as algorithms, into a completely different area of mathematics, such as field theory. This was applied to transform the Collatz Conjecture, an unsolved problem about integers, into a statement about topology (a field of math related to geometry), which allowed the solution of a class of problems similar to the Collatz Conjecture. This was also extended to produce results in computer science, most notably a new method of generating problems unsolvable by computers as well as an algorithm allowing computers to perform the transformations between areas of math in certain areas.

Conclusions/Discussion
This research provides a powerful new tool for mathematical work and makes significant progress on the Collatz Conjecture, and has applications throughout the field of mathematics and computer science. One practical application is the project#s usefulness in Artificial Intelligence, particularly the in the theory of Automated Theorem Provers, which underlie the safety of NASA craft and other systems.

Summary Statement
This project provides a method of transforming problems from one area of mathematics to others, allowing tools from one area of math to be used to solve problems in other areas.

Help Received
Teacher helped with entry and science fair procedures, Dr. David Roe (PIMS fellow at the University of Calgary) helped review the final result for accuracy (no research was done at the university or by Dr. Roe, he only reviewed the final result)