



**CALIFORNIA STATE SCIENCE FAIR
2015 PROJECT SUMMARY**

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Project Title
New Results on the Genus of Complete Graphs with Excised Edges and Parameterization of the Resulting Isomorphism Classes

Abstract

Objectives/Goals
This research project explores the effect on the genus of a complete graph K_v on v vertices when edges are removed and also explores how to classify the resulting graphs. The genus is an important characteristic of a graph, providing a single numerical measure of the complexity of the graph. Also of importance in graph theory is K_v , which is universal in the sense that all graphs are subgraphs of some K_v . Thus, understanding how the genus of K_v responds to the excision of edges is of major importance in graph theory.

Methods/Materials
Three major mathematical fields were used to study the genus of the graphs resulting from excising edges of K_v . Low dimensional topology was used to link the genus of a graph to surface theory, graph theory was used to establish bounds on the genus, and combinatorics were used to calculate the number of isomorphism classes of graphs obtained from K_v when edges are excised.

Results
Three major new results were established. The first considers how many edges can be removed from K_v without changing its genus. I was able to give a sharp answer to this problem by introducing a function $F(v) = (v-2)(v-5)/2 \pmod{6}$. Then my first main result is that if $F(v)$ or fewer edges are removed from K_v , then the genus of the resulting graph is the same as the genus of K_v . My second result is a general result, independent of the genus result, that parameterizes the isomorphism classes of graphs of the form $K_v - h$ edges. This result states that these isomorphism classes are parameterized by the isomorphism classes of graphs with h edges, both connected and non-connected, and with no vertices of degree 0. My third result combines my first two results to parameterize the isomorphism classes of graphs of the form $K_v - F(v)$ edges, which are then calculated. Thus this result explicitly classifies graphs derived from K_v by the removal of $F(v)$ edges. By my first result, these graphs then have the same genus as K_v . Applications of these results to complex networks, such as social networks and the internet, are also given.

Conclusions/Discussion
This project gives new results in graph theory related to the stability of the genus of K_v with respect to the excision of edges. The parameterization of the isomorphism classes of the resulting graphs is calculated. These results can be extended in two directions, to other well-known classes of graphs and to the excision of edges that decrease the genus of K_v .

Summary Statement
This research project presents new results on the stability of the genus of a complete graph K_v with respect to the excision of edges and gives an explicit parameterization of the resulting isomorphism classes of graphs of the form $K_v - F(v)$.

Help Received
My mother helped me build the various physical models that I used to illustrate my results.