



CALIFORNIA STATE SCIENCE FAIR
2015 PROJECT SUMMARY

Name(s) Timothy J. Varghese	Project Number S1430
Project Title From Sums over Natural Numbers to Sums over Primes	
<div><div>Objectives/Goals<p>Any sum over the naturals such as $1 + 2 + 3 \dots n = F(n)$ can be converted to a sum over the primes such as $2 + 3 + 5 + \dots p_n = P(n)$ where p_n is the n-th prime. I started by proposing an Expand-Sum-Prune (ESP) heuristic in which $P(n)$ is approximated as $F(n \ln n) / \ln n$. ESP provides correct asymptotic results for sums of prime powers, duplicating a result of Salat-Znam. The goals of this project are:</p><ol style="list-style-type: none">1. To examine the hypothesis that ESP fails when any one term is too large a fraction of the whole sum.2. To find new sums over primes never published earlier3. When ESP fails, to find better summation methods.</div><div>Abstract</div><div>Methods/Materials<ol style="list-style-type: none">1. Series: I studied several sums over primes including the alternating series $(2 - 3 + 5 - 7 + \dots)$, reciprocal sums $(1/(2*3*5) + 1/(3*5*7) + \dots)$, and sums of prime powers $(2^2 + 3^2 + 5^2 + \dots)$2. All estimates were checked for accuracy using a Visual C program that uses the sieve of Eratosthenes to produce (and sum) all primes up to 1000000.</div><div>Results<ol style="list-style-type: none">1. Alternating sum: Consider $A = 2 - 3 + 5$. I provide a new estimate of $A = 0.5 p_n$ with errors less than 2% for $500 < n < 78,401$ by summing half the prime gaps using a modified ESP method. When my estimate was posed on Math Overflow (viewed 386 times, 11 votes +1 badge for "good question"), mathematicians felt my new result was "almost certainly true". However, using current sieve techniques they can only prove unconditionally that $A < p_n / 64$. My method generalizes to alternating series of prime powers. I published a new series for alternating primes squared in the Online Encyclopedia on Integer Sequences (OEIS) as A240860.2. Reciprocal sums: I prove that $S = 1/(2*3*5) + 1/(3*5*7) + \dots$ converges and $0.0474 < S < 0.0475$, published in the OEIS as A242187 using a Bound-Reduce that applies to the infinite series for e.3. Better estimates for sums of prime powers: I found a better approximation than the Salat-Znam estimate using a balancing constant c. I found experimentally that the best values of c are roughly 0.6 for prime sums, 0.7 for squared sums, and 0.9 for cubed sums4. New estimates from old: I found a new asymptotic estimate for prime products two at a time with 2% error, added to the OEIS as A024447.</div><div>Conclusions/Discussion<p>The hypothesis that ESP method fails if any term dominates (limit of ratio of largest term to sum does not tend to zero) is supported by results.</p></div></div>	
Summary Statement <p>As in Alladin where the peddler promises new lamps for old, I seek new series over primes from old series over integers, and new formulas derived from formulas for integers</p>	
Help Received <p>Neil Sloane, head of OEIS helped refine hypothesis, Erich Bach (Wisconsin) helped make program efficient, Father helped with program. Robert Oliver (Stanford) gave valuable suggestions.</p>	