



**CALIFORNIA STATE SCIENCE FAIR
2017 PROJECT SUMMARY**

Name(s) Andrei Mandelshtam	Project Number J1505
Project Title Limiting Behavior of the Iterations of Tangent	
Abstract Objectives/Goals The objective of this project was to study the statistical properties of the iterations of $\tan(x)$. Methods/Materials Laptop computer with Fortran compiler and Grace. Ran program on 100 initial values of x , iterating tangent on each one 10 million times, and distributed the iterations in a histogram. Results Two new theorems were proven. The first theorem showed that the iterations of $\tan x$ have no limit unless some iteration falls directly into a solution to $\tan(x)=x$, this being only a countable number of points. My most interesting theorem is that periodic points are dense on the real number line. Additionally, the inverse square law was discovered numerically by studying the distribution on a histogram of the first 10 million iterations of $\tan(x)$ for 100 different starting points y . It was found that each of these histograms closely resembled the graph of C/x^2 for a varying constant C . The values of C computed by the method of least squares showed random-like behavior. Then each histogram was scaled by its value at a fixed point a . It was found that each one resembled the same graph, a^2/x^2 , no matter what starting point y . Conclusions/Discussion Through rigorous and numerical results, it was shown that the iterations of tangent have a very interesting structure with many unusual patterns and properties. Two theorems describing the iterations of $\tan(x)$ were proven rigorously using the methods of calculus, trigonometry, and mathematical induction. Also, it was shown numerically that the iterations have a universal, non-random distribution.	
Summary Statement I studied the iterations of the tangent function, proving two theorems and discovering a universal law their distribution satisfied.	
Help Received My father taught me programming in Fortran. My mother gave me ideas on what to investigate after I proved my first theorem. My science teacher, Mrs. Martin, gave me general Science Fair advice.	