Name(s)

Espen Slettnes

Project Number

J1507

Project Title

Minimal Embedding Dimensions of Rectangle k-Visibility Graphs

Abstract

Objectives/Goals

Research on bar visibility graphs was originally motivated by problems about constructing VLSI (Very Large Scale Integration) circuits, and were adopted in the 1980s as a geometric model to represent traces, e.g. on circuit boards and in VLSI chip designs.

Rectangle visibility graphs were introduced by Bose et al in 1997 as a generalization of bar visibility graphs. A graph is a rectangle visibility graph if it can be represented with vertices as disjoint axis-parallel rectangles, such that there is an unobstructed axis-parallel line of sight between two rectangles if and only if there is an edge between the corresponding vertices.

I combined rectangle visibility graphs with k-visibility to form rectangle k-visibility graphs, in which the line of sight between two rectangles in the representation can be obstructed by at most k other rectangles.

I then took a natural generalization of rectangle k-visibility graphs into higher dimensions. I found that given enough spacial dimensions there exists a rectangle k-visibility representation of any graph G. I continued to study its properties, and proceeded to bound it for complete graphs, complete r-partite graphs, and hypercube graphs.

Results

I established upper bounds on the number of dimensions needed to represent the above types of graphs as rectangle k-visibility graphs, in some cases with the added restriction that the rectangles be unit rectangles, and/or that k=0. Additionally I established a similar upper bound on the minimal embedding dimension on the Cartesian product of multiple graphs.

Conclusions/Discussion

The representation of graphs as hyper-rectangles with k-visibility lines is an exciting extension of existing visibility graph concepts, and like previous work in the field, is likely to have applications not yet imagined. In future research I hope to sharpen the bounds presented here, to study additional types of graphs, and to study different types of visibility.

Summary Statement

I explored the number of dimensions required to represent various graphs as hyperrectangles with axis-parallel visibility lines.

Help Received

Through a MIT PRIMES-USA research internship, I received mentorship from Dr. Jesse Geneson, who introduced me to the topic of visibility graphs.